The two-loop QCD amplitude $gg \to h, H$ in the Minimal Supersymmetric Standard Model

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We present the two-loop QCD amplitude for the interaction of two gluons and a CP-even Higgs boson in the Minimal Supersymmetric Standard Model. We apply a novel numerical method for the evaluation of Feynman diagrams with infrared, ultraviolet and threshold singularities. We discuss subtleties in the ultraviolet renormalization of the amplitude with conventional dimensional regularization, dimensional reduction, and the four dimensional helicity scheme. Finally, we show numerical results for scenarios of supersymmetry breaking with a rather challenging phenomenology in which the Higgs signal in the MSSM is suppressed in comparison to the Standard Model.

The loop-mediated interaction of a Higgs boson and gluons is the main production mechanism for a Higgs boson at hadron colliders. Depending on the decay channels in which a Standard Model Higgs boson may be discovered, the measurement of the signal cross-sections can be achieved with a precision of about $\pm 10\%$ or better [1]. Amid the discovery of a Higgs boson, the signal cross-section will be an independent precision test of the Standard Model and its extensions.

The gluon-fusion Higgs boson production cross-section is sensitive to higher order QCD corrections [2, 3, 4]. At the LHC, the signal cross-sections are known to change up to a factor of two when perturbative corrections from $\mathcal{O}\left(\alpha_s^2\right)$ through $\mathcal{O}\left(\alpha_s^4\right)$ are included [5, 6]. The accuracy of these theoretical predictions is about $\pm 10\%$. The partonic decay width to gluons also increases by a factor of two when including the known higher order QCD corrections. This quantity is now computed through order $\mathcal{O}\left(\alpha_s^5\right)$ with an accuracy better than 1% [7]. Theoretical uncertainties in the ggh interaction within the Standard Model are currently adequately small for a future comparison with LHC data at a 10% precision level.

Extensions of the Standard Model (SM) postulate diverse mechanisms for breaking the electroweak symmetry. A different Higgs boson sector than the one in the Standard Model and additional new particles are often introduced. The effects of undiscovered particles on the gluon-fusion cross-section are rather unconstrained from experimental data. Novel colored particles can change the Higgs and gluon interaction dramatically. For example, an additional heavy-quark with SM-like Yukawa coupling to the Higgs boson would contribute almost as much as the top-quark to the ggh amplitude. Recent examples in well motivated models were shown in [8].

The complexity of the two-loop SM computations at $\mathcal{O}\left(\alpha_s^3\right)$ in the full theory and at $\mathcal{O}\left(\alpha_s^4\right)$ in the limit of a heavy top-quark is serious. The methods that have been employed are powerful and may be employed in other models. In especially simple modifications of the SM Lagrangian, for example adding a fourth generation with heavy leptons and quarks, the existing calculations are already sufficient. However, it will be important to know the gluon-fusion cross-section through at least order $\mathcal{O}\left(\alpha_s^3\right)$ in all models which aspire to explain LHC data. Many viable extensions of the SM contain colored particles where, at two-loops, more than one of these massive particles appears in diagrams contributing to the gluon-gluon-higgs amplitude. Known analytic methods for two-loop calculations are restricted to problems with a small number of mass parameters. New techniques to evaluate multi-loop integrals with diverse mass-scales are indeed required.

In this article, we compute the two-loop QCD amplitude $gg \to h, H$ for a CP-even light (h) and heavy (H) Higgs boson in the minimal supersymmetric extension of the Standard Model (MSSM). We employ a numerical method which we have recently developed for multi-loop calculations [9, 10]. In our method, Feynman diagrams with diverse combinations of massive and massless propagators are treated on an equal footing. Given the complexity of the MSSM, the computation of $2 \to 1$ two-loop amplitudes in other extensions of the SM may also be tractable.

Partial SM-like contributions from supersymmetric diagrams with only squarks in the loops have been recently computed in the literature [11, 12, 13, 14]. The MSSM amplitude is also known in the limit of a light Higgs boson with respect to quarks, squarks, and gluinos [15, 16, 17].

The computation of the two-loop amplitude without using an effective theory approach is well motivated in SM extensions. Relatively light colored particles (lighter than the the top-quark) are not excluded experimentally. Heavy Higgs bosons are also predicted in the spectrum of new theories. Also, the couplings of the bottom quark to Higgs bosons may be significantly enhanced in models with more than one Higgs doublet. The MSSM exhibits all of these features; knowledge of the amplitude for the gluon-gluon-Higgs interactions without assumptions about the mass hierarchy of Higgs bosons and colored particles is therefore especially important. We consider the MSSM an archetype for many other models regarding its computational challenges. To the best of our knowledge, we present here the first complete result for a two-loop three point Green's function in the MSSM.

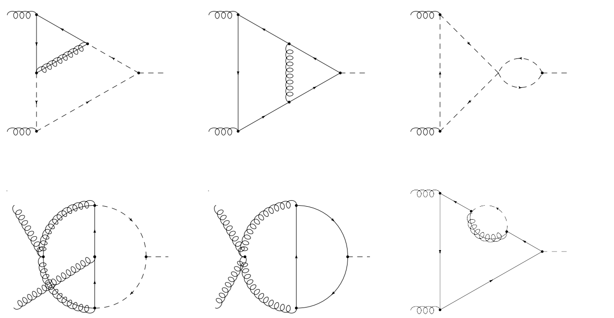

FIG. 1: Sample of two-loop diagrams containing up to four different mass parameters in the propagators.

The $gg \to h$ amplitude at $\mathcal{O}\left(\alpha_s^3\right)$ includes 135 two-loop Feynman diagrams. We have generated them using QGRAF [18]. We implemented the MSSM Feynman rules following the method of Ref. [19] for Majorana fermions. Traces of Dirac matrices and the color algebra are carried out with programs written in standard algebraic manipulation packages, including FORM [20]. We have checked that our computer programs and FeynArts [21] generate equivalent integrands for the amplitude.

We depict some of the Feynman diagrams in Fig. 1. We computed diagrams with only one massive particle in the loops both analytically [11] and numerically [9]. We computed the remaining diagrams numerically applying the method of [9, 10]. We review here only the salient features of the numerical method.

We combine propagators in each loop with a set of Feynman parameters and integrate out the loop momenta. The Feynman parameters are mapped to a unit hypercube integration domain. At a second step, we apply a sector decomposition algorithm [22, 23] to factorize divergences in the Feynman parameters which are regulated by the dimensional parameter $D = 4 - 2\epsilon$. We deform the contour of integration away from the poles which may occur for physical values of the external momenta and mass parameters [24]. Finally, we extract the factorized singularities in ϵ with independent subtractions on the real part of Feynman parameters and perform an expansion around $\epsilon = 0$ [9]. With this procedure the amplitude is written in terms of $\mathcal{O}(1000)$ integrals which can be evaluated with standard numerical methods. We have used the Divonne and Cuhre integration algorithms from the Cuba library [25].

When we apply this method naively to diagrams such as the last diagram in Fig. 1 with a sbottom-gluino bubble inserted in a bottom propagator, the numerical integration shows a very poor convergence. Our technique is based on a deformation of Feynman parameters:

$$x_i \to z_i = x_i + i\lambda_i y_i$$

The function y_i is constructed using the prescription of [9, 24], and it depends on Feynman parameters and the mass parameters. We can usually set a common numerical value for the constants λ_i, which dictate the magnitude of the deformation, when all mass values are of the same order of magnitude. However, the large hierarchy of the squared mass values for the bottom quark and the gluino or the bottom squarks may yield y_i's of disparate magnitude for different Feynman parameters. Small values of y_i may not distance the contour of deformation sufficiently from singularities. In turn, large y_i values may result to a contour which is not equivalent with the integration over the original Feynman parameters. We can remedy these problems by adjusting the parameters λ_i independently for each Feynman parameter. We perform a first Monte-Carlo integration with a small number of integrand evaluations setting all λ_i to a common value λ. We use this integration in order to obtain an estimate of the maximum values y_i^{max} of y_i. We then compute the integrals setting $\lambda_i \sim \lambda / y_i^{max}$, where λ has a typical value $0.1 - 0.5$. With this refined selection of the parameters which determine the deformation of the integration contour we are able to evaluate all diagrams efficiently.

We were also able to compute the problematic diagrams with a uniform selection of λ_i values by using a Feynman parameterization similar to the one in Ref. [23]. This parameterization casts diagrams with bubble subgraphs as a sum of two terms. One of them corresponds to an one-loop integral and matches to the counter-term for mass renormalization. The second term corresponds to a two-loop integral which can be evaluated without a specially tuned contour deformation.

We have used conventional dimensional regularization (DREG) [26], dimensional reduction (DRED) [27, 28] and the four dimensional helicity scheme (FDH) [29] with minimal subtraction. In all schemes an anti-commuting γ_5 prescription was employed. Similarly to [15], we decouple the top-quark, squarks and gluino from the running of the strong coupling α_s. In addition, we perform a pole renormalization of all masses and renormalize the squark higgs couplings as in [30].

DREG is known to violate the symmetries of the MSSM. DRED is a consistent renormalization scheme for the computation of the MSSM amplitude through the next to leading order in the strong coupling. The two schemes are, however, related by finite shifts in the coupling constants and masses [32]. This equivalence only holds when new terms are added to the Lagrangian in DRED [32, 33, 34]: operators with ϵ-scalars which arise when the 4-dimensional gluon field is split into a D-dimensional part and its remaining $4 - D$ components. Notably, in the MSSM, the only relevant term for the amplitude $gg \to h$ is the mass term for the ϵ-scalars. As this mass can be always absorbed into a redefinition of the squark masses [35], the calculation in the MSSM can be performed by setting it to zero.

The picture described above changes dramatically when considering a theory with less symmetries than the

MSSM, where supersymmetry is only softly broken and the possible ϵ-scalar couplings are still very restricted. We observed that, if either $SU(2)_L$ or (softly broken) supersymmetry are absent, a new coupling between the ϵ-scalars and the higgs fields emerges radiatively. This coupling is indispensable in order to render the one-loop cross-section for the process $h \to \epsilon\epsilon$ to be of $\mathcal{O}(\epsilon)$ (evanescent). In the MSSM this happens automatically due to a cancellation among contributions from up and down type quarks and the corresponding squarks. In the SM this cancellation does not take place. Most importantly, the ϵ-higgs coupling and its consistent renormalization should be included in order for the results for the SM two-loop amplitude $gg \to h$ in DRED and DREG to agree.

We compared the results in the two schemes after we accounted for known shifts in the strong coupling α_s and the mass parameters between the two schemes. We found that the two results agree only if an additional shift in the higgs-squark-squark coupling is performed. We find that the relation between the renormalized coupling $m_q^2 V_{h\tilde{q}_i\tilde{q}_j}$ in DREG and DRED is,

$$\left(m_q^2 V_{h\tilde{q}_i\tilde{q}_j}\right)^{\rm DREG} - \left(m_q^2 V_{h\tilde{q}_i\tilde{q}_j}\right)^{\rm DRED} = \frac{\partial\left(m_q^2 V_{h\tilde{q}_i\tilde{q}_j}\right)}{\partial m_q}\left[\delta_{m_q}^{\rm DRED} - \delta_{m_q}^{\rm DREG}\right] + \mathcal{O}\left(\alpha_s^2\right), \quad (1)$$

where $V_{h\tilde{q}_i\tilde{q}_j}$ is the dimensionless part of the tree coupling, and $\delta_{m_q}^{\rm DRED}$,$\delta_{m_q}^{\rm DREG}$ are the pole mass-renormalization counter-terms in the two schemes. We have verified that the shift in Eq. 1 is also needed for the computations of the decay rate $h \to \tilde{q}_i\tilde{q}_j$ to agree in DRED and DREG at one loop. The reason for this shift is simple. The one loop corrections to the decay amplitude $h \to \tilde{q}_i\tilde{q}_j$ are identical in DRED and DREG before renormalization. However, the renormalization scheme for the squark higgs coupling in [30] involves the quark pole mass, which is related to the bare mass trough a scheme dependent expression. The shift in Eq. 1 simply cancels that dependence in such a way that the DRED result for the decay rate is recovered.

We can avoid to compute the contribution of operators with ϵ-scalars to the $gg \to h$ amplitude for the MSSM by using the FDH scheme. DRED and FDH treat external polarizations differently; this gives rise to a relative factor of $(1-\epsilon)^2$ in the two-schemes for the squared amplitudes through $\mathcal{O}(\alpha_s^3)$. Diagrams with internal ϵ scalars in DRED are accounted for in the FDH scheme by diagrams with internal D_S-dimensional gluons, where D_S is the dimension of the spin algebra. The dimensionality of the loop integrals D is kept distinct with $D < D_S$; after performing the loop-integrations an analytic continuation of D_s to 4 dimensions [29] takes place. The FDH scheme cannot account for the contributions of diagrams with the ϵ-higgs coupling. As we discussed, these are needed for the $gg \to h$ two-loop amplitude in less symmetric theories than the MSSM, such as the SM. The discrepancy cannot be absorbed in any coupling or mass redefinition, and the FDH result is inconsistent with the results in DRED and DREG for the SM two-loop amplitude.

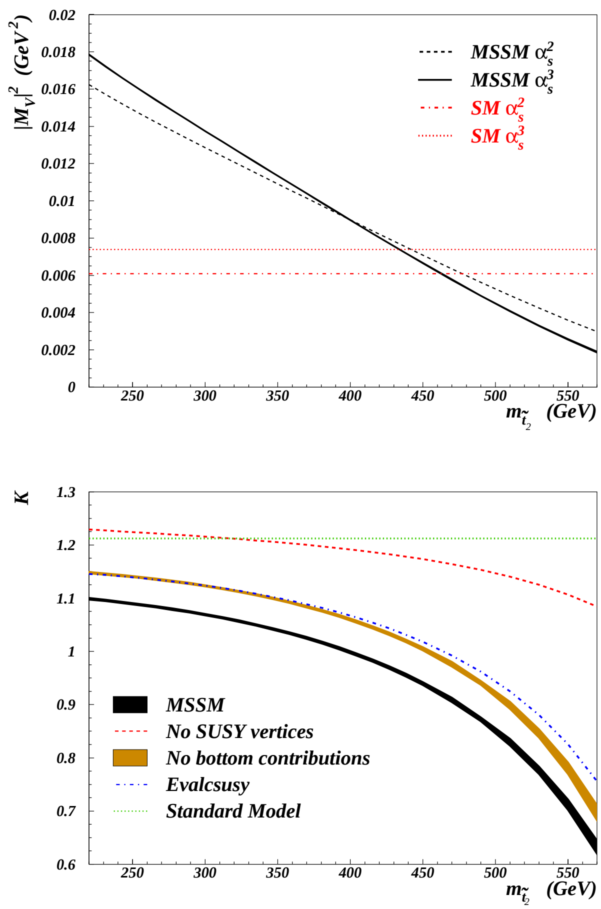

FIG. 2: The squared UV and IR renormalized amplitude for a light Higgs boson through $\mathcal{O}(\alpha_s^3)$ and the corresponding K-factors

We now present numerical results for the MSSM two-loop amplitudes $gg \to h, H$. We neglect here the Higgs couplings to quarks and squarks other than the ones in the third generation. The two-loop amplitudes are infrared divergent with poles up to second order in the dimension parameter ϵ. The singular part is universal [36] and cancels against other universal contributions at the same order in α_s from real radiation processes. We present here the finite part after UV renormalization in the $\overline{\rm MS}$ scheme and subtracting the infrared counterterm of Ref. [37]. A complete phenomenological analysis requires the inclusion of the non-singular parts from real radiation and will be the subject of a future publication. Nevertheless, useful conclusions may also be inferred from solely the two-loop amplitudes since they include all diagrams with more than one massive inter-

nal particle. These are the diagrams which had not been computed earlier in the literature [11, 12, 13, 14] and they form a subset which is infrared finite.

In Figure 2 we present our results for the production of a light higgs in gluon fusion in the MSSM. On the upper panel we show the renormalized squared amplitude for a light neutral Higgs boson $gg \to h$ through order $\mathcal{O}(\alpha_s^3)$, averaging over external gluon polarizations and colors. As discussed above, we subtracted a universal infrared counter-term in order to obtain a finite result. We show the SM value as a reference. The lower panel shows the corresponding K-factor: the ratio of the squared amplitude through $\mathcal{O}(\alpha_s^3)$ divided by the $\mathcal{O}(\alpha_s^2)$ result. We also include the K-factors obtained in various approximations of our full result. We write our results in terms of the $\alpha_s^{\overline{\rm MS}}$ with $\alpha_s^{\overline{\rm MS}}(M_Z) = 0.1176$. We choose a relatively small mass value for one of the stop squarks $m_{\tilde{t}_1} = 150\,\text{GeV}$ and vary the mass of the heavy stop quark $m_{\tilde{t}_2}$. In the bottom sector we have set $m_b = 5\,\text{GeV}$, $m_{\tilde{b}_1} = 350\,\text{GeV}$ and $m_{\tilde{b}_2} = 370\,\text{GeV}$. For the SUSY parameters, we have chosen $\alpha = 3^\circ$, $\tan\beta = 20$, $\mu = 300\,\text{GeV}$, and the squark mixing angles $\theta_{\tilde{t}} = \theta_{\tilde{b}} = 40^\circ$. Finally, $m_{\text{gluino}} = 500\,\text{GeV}$, $m_h = 115\,\text{GeV}$, the renormalization scale is fixed to the higgs mass, $\mu_{\text{ren}} = m_h$. The renormalization of the squark mixing angles is done as in [15] at a scale $\mu_\theta = 200\,\text{GeV}$.

The squared amplitude in the MSSM decreases for growing $m_{\tilde{t}_2}$. This is due to large cancellations between diagrams where a top or a light stop couple to the Higgs boson. The $\mathcal{O}(\alpha_s^3)$ contribution ranges from 15% to -40%, becoming negative at large values of $m_{\tilde{t}_2}$. A significant part of this correction originates from the infrared finite subset of diagrams with gluinos and squark quartic couplings; their contribution to the squared amplitude is negative and grows in absolute value with growing $m_{\tilde{t}_2}$. The contribution from bottom and sbottom loops is below 3% for this value of $\tan\beta$, and smaller for lower values of $\tan\beta$. We also show the K-factor obtained with the effective theory calculation of Ref. [15] using the published program `evalcsusy`. The approximation given by this calculation is remarkably good. At small values of $m_{\tilde{t}_2}$ it almost coincides with the MSSM result when neglecting the bottom contributions whereas at larger values of the mass splitting it differs only by a few percent. The growing (small) discrepancy is due to the contribution of the heavy stop decreasing with a larger mass $m_{\tilde{t}_2}$ rendering the contribution of light stop loops, which are not perfectly approximated by the effective theory, more significant.

The scenaria with a light scalar quark have received recent attention. In particular, a large mass gap in the two scalar-top physical states reduces the "fine-tuning" of the MSSM [38]; this may be an outcome of supersymmetry breaking via "mirage" mediation [39]. In Fig 2 we observe that the squared amplitude decreases as the squark mass splitting increases. The parameter region where both stop squarks are light is excluded by measurements of the $\rho-$parameter [38]. However, as the stop mass difference increases we obtain viable parameter values; they result to a ggh interaction which is significantly weaker than in the SM; the discovery of a light Higgs boson may then become very difficult at the LHC.

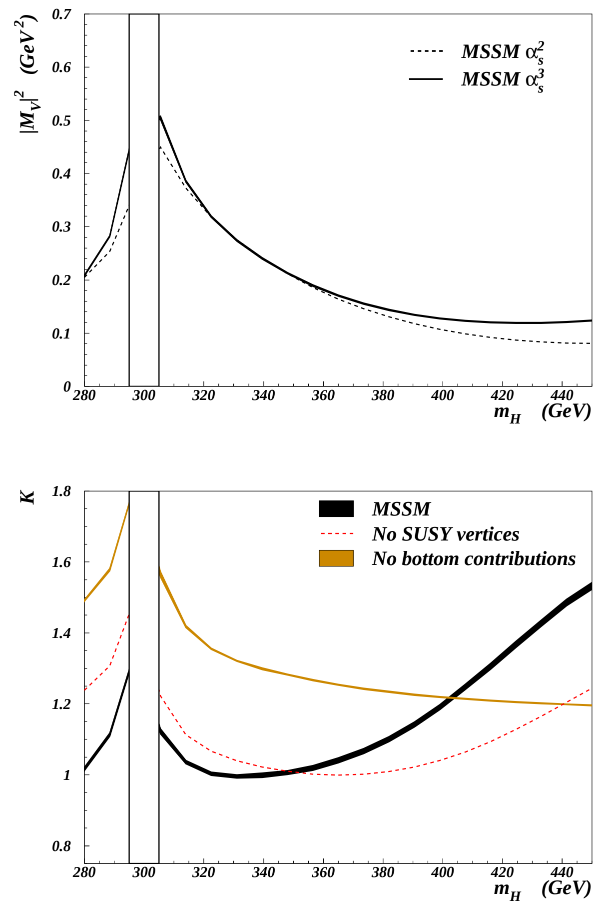

FIG. 3: The squared UV and IR renormalized amplitude for a heavy neutral Higgs boson through $\mathcal{O}(\alpha_s^3)$ and the corresponding K-factors

In Figure 3 we show the corresponding results for the production of a heavy Higgs. In this case we fixed the heavy stop mass $m_{\tilde{t}_2} = 350\,\text{GeV}$ and varied the heavy Higgs boson mass. We set the renormalization scale to $\mu_{\text{ren}} = m_H$. All the other masses and parameters are identical to the light Higgs boson calculation described above. At $m_H = 2\,m_{\tilde{t}_1} = 300\,\text{GeV}$ there is a threshold, where the perturbative calculation diverges. In the plots, we have conservatively whitewashed a window of 5 GeV around the threshold, but we have checked that our numerical calculation works fine for phase space points much closer to this threshold. We find that the $\mathcal{O}(\alpha_s^3)$ corrections in the MSSM are very mild, growing to

about 20% in the region of the threshold but ammounting to only a few percent anywhere else. The bottom sector contributions are, however, very important, ammounting to almost 40% for small Higgs boson masses. As in the light Higgs boson case, contributions from the diagrams with gluinos and squark quartic couplings are substantial.

In this paper we have computed the full two-loop amplitudes $gg \to h$ and $gg \to H$ in the MSSM, a complicated extension of the Standard Model. We have developed a powerful new method for multi-loop calculations and applied it to compute the first two-loop three-point amplitude known in the MSSM for arbitrary masses of sparticles. Our results will improve the precision of cross-section predictions for the gluon-fusion process in the MSSM. We are looking forward to further applications of our method.

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